Suspensions of viscoelastic capsules: effect of membrane viscosity on transient dynamics

Fabio Guglietta[1], Francesca Pelusi[2], Othmane Aouane[3], Marcello Sega[4], Jens Harting[3,5]

1 Tor Vergata University of Rome, Dept. of Physics & INFN (IT)
2 Italian National Research Council (CNR), Institute for Applied Mathematics (IT)
3 Helmholtz Institute Erlangen-Nürnberg for Renewable Energy (IEK-11), Dept. of "Dynamics of Complex Fluids and Interfaces" (DE)
4 University College London (UCL), Dept. of Chemical Engineering (UK)
5 Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Dept. of Chemical and Biological Engineering & Dept. of Physics (DE)

Motivations: why viscoelastic capsules are important?

TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Biomedical applications

Pharmaceutical industry

Cosmetic

Food processing

- Designing some **biomedical devices** (e.g., heart pumps)
- **Reduced models** to estimate the **rate of hemolysis** that take the **mechanical properties** of red blood cells (RBCs) as input
- Characterise the **deformation** of RBCs and the **time** they take to **deform**

KEYWORDS
Viscoelastic membrane
Deformation
Deformation time

Effects of membrane viscosity μ_m on RBC dynamics

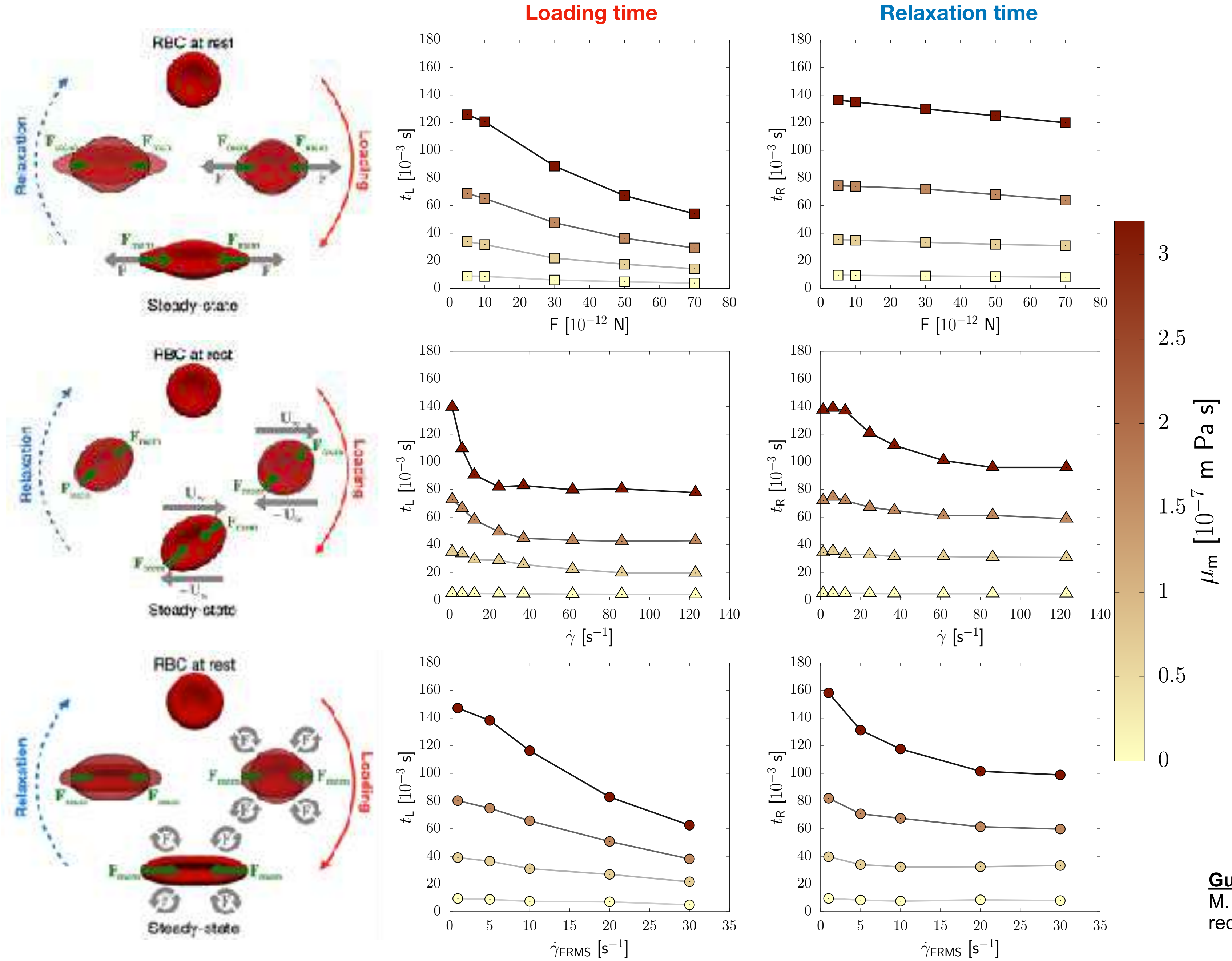
- Increasing μ_m results in an **increase of the loading time** t_L [1,2]
- Depending on the flow conditions even the steady value of the **deformation depends on** μ_m [1,2]

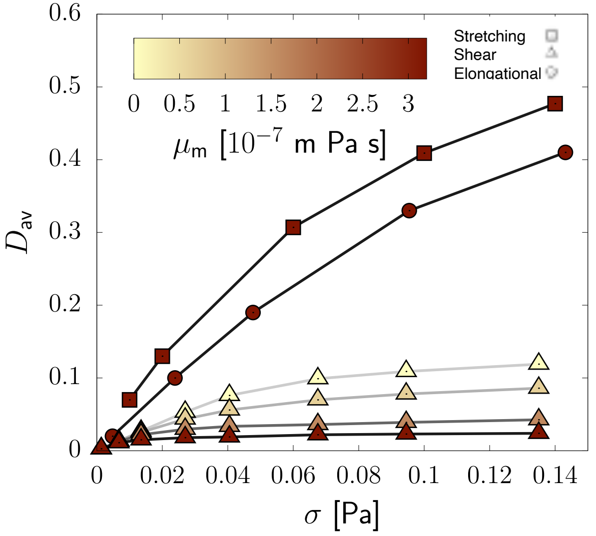
(1) **Guglietta, F.,** Behr, M., Biferale, L., Falcucci, G., & Sbragaglia, M. (2020). On the effects of membrane viscosity on transient red blood cell dynamics. Soft Matter, 16(26), 6191-6205.
(2) **Guglietta, F.**, Behr, M., Falcucci, G., & Sbragaglia, M. (2021). Loading and relaxation dynamics of a red blood cell. Soft Matter, 17(24), 5978-5990.

Stretching with optical tweezers

Simple shear flow

Elongational flow





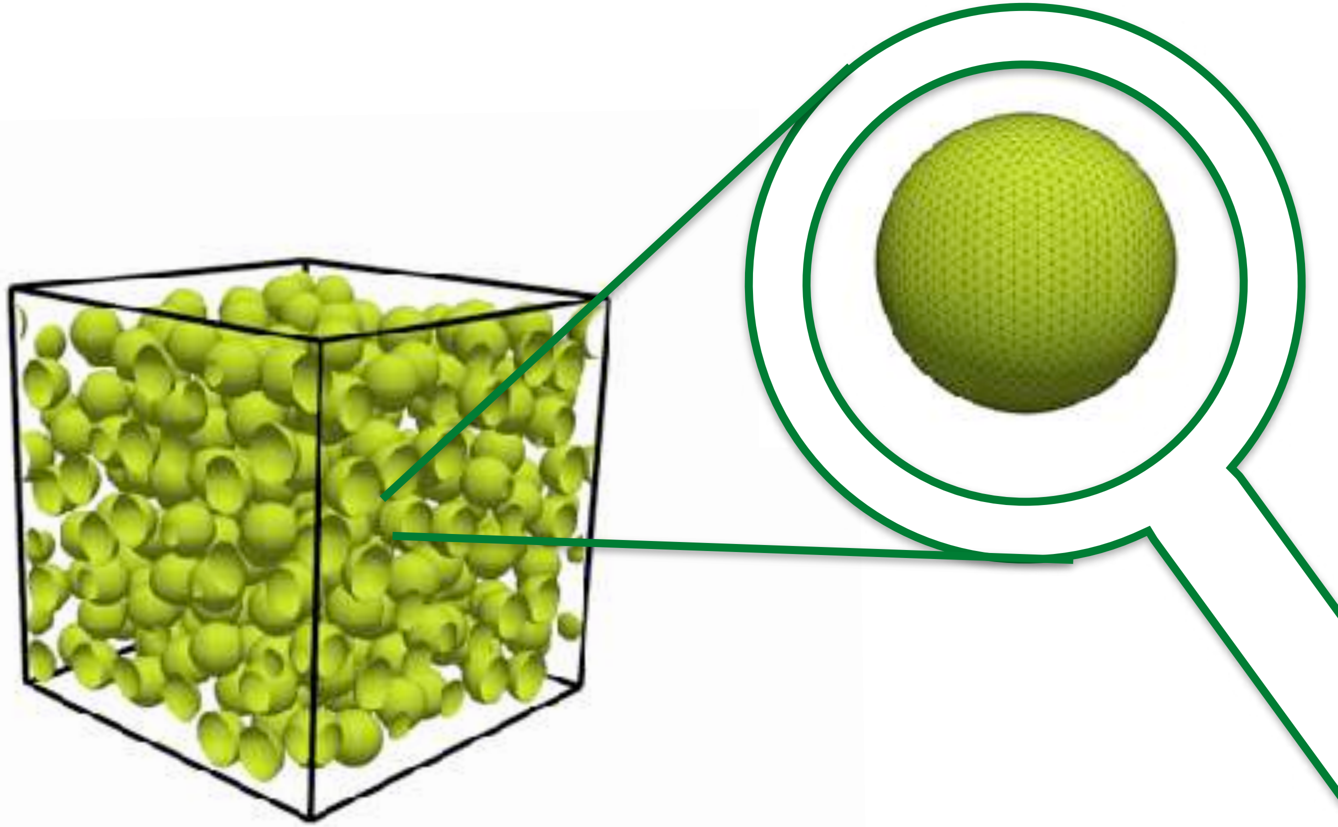
Guglietta, F., Behr, M., Falcucci, G., & Sbragaglia, M. (2021). Loading and relaxation dynamics of a red blood cell. Soft Matter, 17(24), 5978-5990.

Outlook

- **Aim:** Characterise the effect of **membrane viscosity** on the mechanical response (i.e., deformation, loading time) of suspensions of viscoelastic spherical capsules.

- **Numerical model:**
  - Viscoelastic **membrane** model
  - **Fluid** solver

- **Results:**
  - **Single** viscoelastic capsule
  - **Suspensions** of viscoelastic capsules



TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Numerical model

Membrane model

TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Energy:

$$W = W_s + W_v$$

Skalak model (resistance against strain) [1,2]

$$W_s = \frac{1}{12}\int \left[k_s \left(I_1^2 + 2I_1 - 2I_2 \right) + k_\alpha I_2^2 \right] dS$$

k_s: Elastic *shear* modulus; k_α: Elastic *dilatational* modulus

Strain invariants:

$$I_1 = \lambda_x^2 + \lambda_y^2 - 2$$
$$I_2 = \lambda_x^2 \lambda_y^2 - 1$$

λ_x and λ_y are the principal stretch ratios

Global volume conservation [2]

$$W_v = \frac{k_v}{2}\frac{(V - V_0)^2}{V_0}$$

Volume modulus: k_v

Initial volume: V_0

Viscous tensor:

$$\tau^v$$

Boussinesq-Scriven law [3,4]

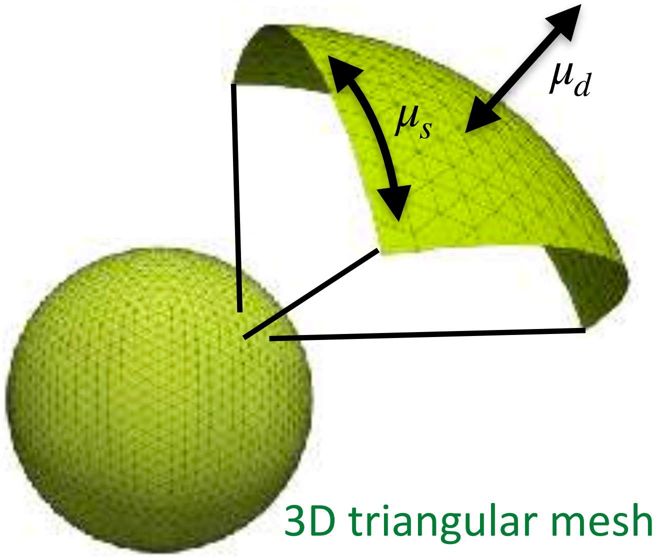
$$\tau^v = \mu_s \left[2\mathbf{E} - \mathrm{tr}(\mathbf{E})\mathbf{I} \right] + \mu_d \mathrm{tr}(\mathbf{E})\mathbf{I} = 2\mu_m \mathbf{E}$$

$$\mathbf{E} = \frac{1}{2}[\nabla \mathbf{u} + \nabla \mathbf{u}^T]$$

μ_s: *Shear* membrane viscosity; μ_d: *Dilatational* membrane viscosity; μ_m: Membrane viscosity

Repulsive force [5]:

$$\vec{\varphi}_{ij} = \begin{cases} \bar{\epsilon} \left[\left(\frac{\Delta x}{d_{ij}} \right)^2 - \left(\frac{\Delta x}{\delta_0} \right)^2 \right] \hat{d}_{ij} & \text{if } d_{ij} < \delta_0 \,, \\ 0 & \text{if } d_{ij} \geq \delta_0 \,, \end{cases}$$



3D triangular mesh (5120 faces)

(1) Skalak, R., Tozeren, A., Zarda, R. P., & Chien, S. (1973). Strain energy function of red blood cell membranes. Biophysical journal, 13(3), 245-264.
(2) Krüger, T. (2012). Computer simulation study of collective phenomena in dense suspensions of red blood cells under shear. Springer Science & Business Media.
(3) Barthès-Biesel, D., & Sgaier, H. (1985). Role of membrane viscosity in the orientation and deformation of a spherical capsule suspended in shear flow. Journal of Fluid Mechanics, 160, 119-135.
(4) Li, P., & Zhang, J. (2019). A finite difference method with subsampling for immersed boundary simulations of the capsule dynamics with viscoelastic membranes. Int. J. Numer. Methods Biomed. Eng., 35(6), e3200.
(5) Aouane, O., Scagliarini, A., & Harting, J. (2021). Structure and rheology of suspensions of spherical strain-hardening capsules. Journal of Fluid Mechanics, 911.

Immersed boundary - lattice Boltzmann (IB-LB) method

TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Lattice Boltzmann (LB) method [1]

Streaming — *Collision* — *Membrane forces*

$$f_i(\mathbf{x}+\mathbf{c}_i\Delta t, t+\Delta t) - f_i(\mathbf{x}, t) = \Delta t \left[\Omega_i(\mathbf{x}, t) + \left(1 - \frac{1}{2\tau}\right) F_i(\mathbf{x}, t) \right]$$

Populations: $f_i(\mathbf{x}, t)$

Density of mass in the physical space and in the (kinetic) velocity space

Fluid density

$$\rho(\mathbf{x}, t) = \sum_i f_i(\mathbf{x}, t)$$

Fluid velocity

$$\rho\mathbf{u}(\mathbf{x}, t) = \sum_i \mathbf{c_i}\ f_i(\mathbf{x}, t)$$

Immersed boundary (IB) method [1]

membrane → fluid

$$\mathbf{F}(\mathbf{x}, t) = \sum_j \varphi_{\mathbf{j}}(t)\Delta(\mathbf{r_j}(t), \mathbf{x}) + \mathbf{F}_{other}$$

fluid → membrane

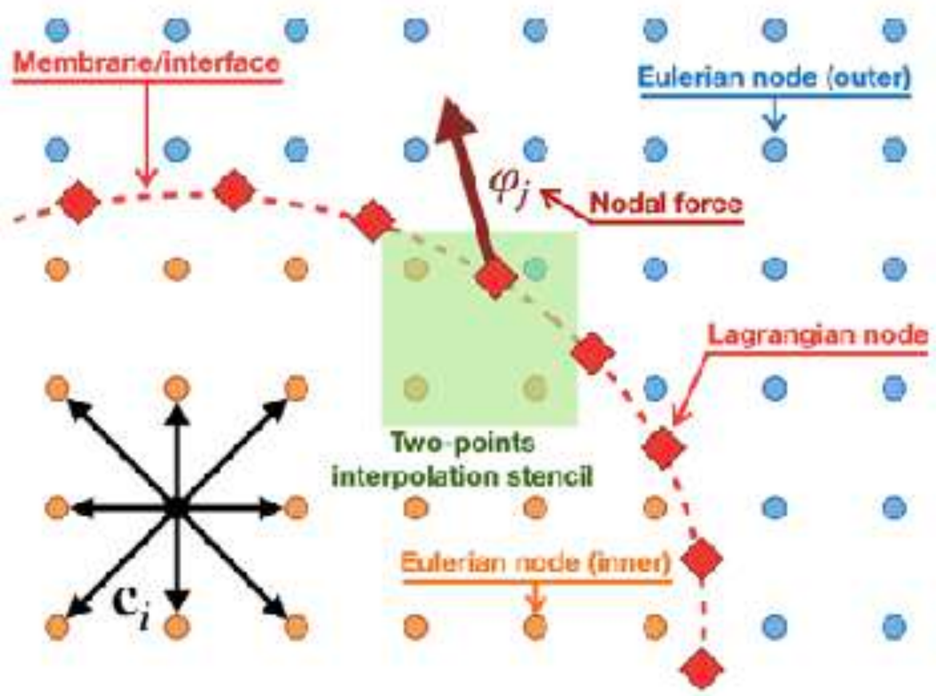
$$\dot{\mathbf{r}}_j(t) = \sum_{\mathbf{x}} \Delta x^3 \mathbf{u}(\mathbf{x}, t)\Delta(\mathbf{r_j}(t), \mathbf{x})$$

$$\mathbf{r}_j(t+\Delta t) = \mathbf{r_j}(t) + \dot{\mathbf{r}}_{\mathbf{j}}(t)$$

Discrete Dirac delta

$$\Delta(\mathbf{x}) = \frac{\Phi(x)\Phi(y)\Phi(z)}{\Delta x^3}$$

$$\Phi_2(x) = \begin{cases} 1 - |x| & 0 \le |x| \le \Delta x \\ 0 & \Delta x \le |x| \end{cases}$$



(1) Krüger, T., Kusumaatmaja, H., Kuzmin, A., Shardt, O., Silva, G., & Viggen, E. M. (2017). The lattice Boltzmann method. Springer International Publishing, 10(978-3), 4-15.

Results: single capsule

Single capsule in simple shear flow

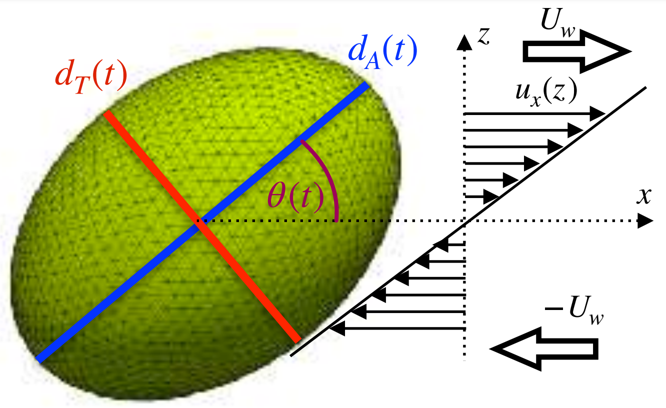
Reynolds number: $\mathrm{Re} = \frac{\dot{\gamma}\rho r^2}{\mu} = 10^{-2}$

Capillary number: $\mathrm{Ca} = \frac{\dot{\gamma}\mu r}{k_s} = \dot{\gamma} t^* \in [0.1,1]$

Boussinesq number: $\mathrm{Bq} = \frac{\mu_m}{\mu r} \in [0,50]$

Radius: r
Shear rate: $\dot{\gamma}$
Fluid viscosity: μ
Membrane viscosity: μ_m
Intrinsic time: $t^* = \mu r / k_s$

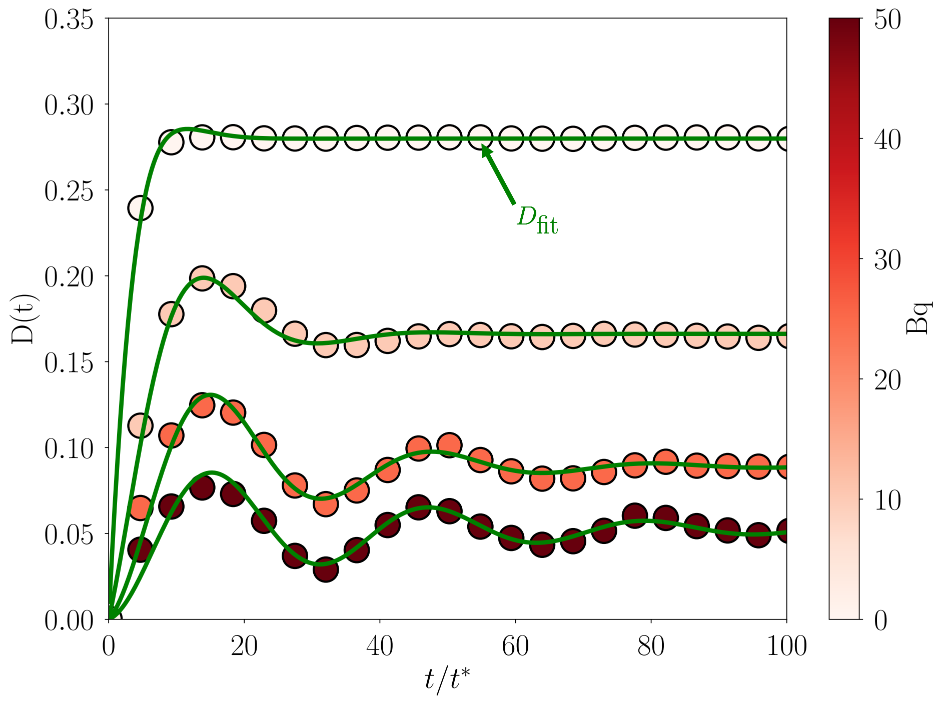
$$D(t) = \frac{d_A(t) - d_T(t)}{d_A(t) + d_t(t)}$$



Fit function:

$$D_{\text{fit}}\left(\frac{t}{t^*}\right) = \bar{D}\left[1 - \exp\left(-\frac{t}{t^* t_L}\right)\cos\left(\omega\frac{t}{t^*}\right)\right]$$

Fit parameters: $t_L,\ \omega$



Bq = 0

Bq = 50

Results: suspensions of capsule

Suspension of viscoelastic capsules

Reynolds number: $\mathrm{Re} = \frac{\dot{\gamma}\rho r^2}{\mu} = 10^{-2}$

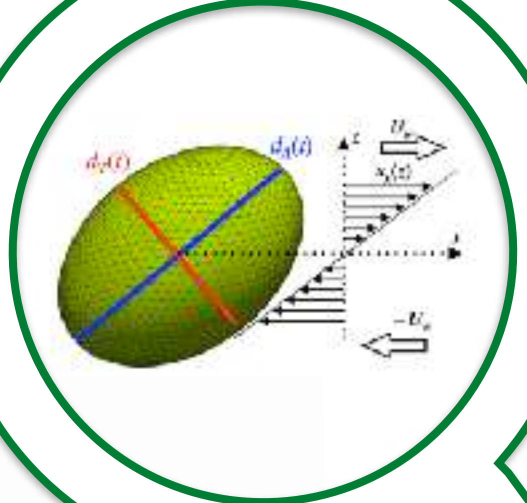
Capillary number: $\mathrm{Ca} = \frac{\dot{\gamma}\mu r}{k_s} = \dot{\gamma}t^* \in [0.1,1]$

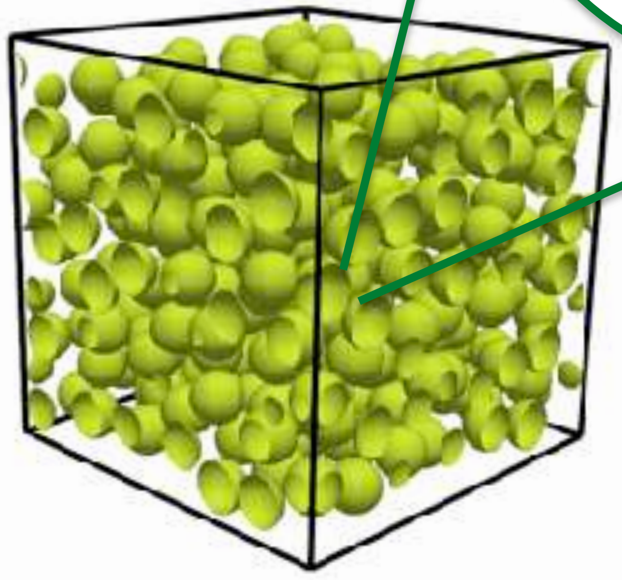
Boussinesq number: $\mathrm{Bq} = \frac{\mu_m}{\mu r} \in [0,50]$

Volume fraction: $\phi = \frac{\sum_i V_i}{L^3} \in [0.001,0.4]$

Deformation: $\langle D \rangle = \frac{1}{N}\sum_i D_i(t)$

Radius: r
Shear rate: $\dot{\gamma}$
Fluid viscosity: μ
Capsule Volume: V_i
Membrane viscosity: μ_m
Intrinsic time: $t^* = \mu r / k_s$





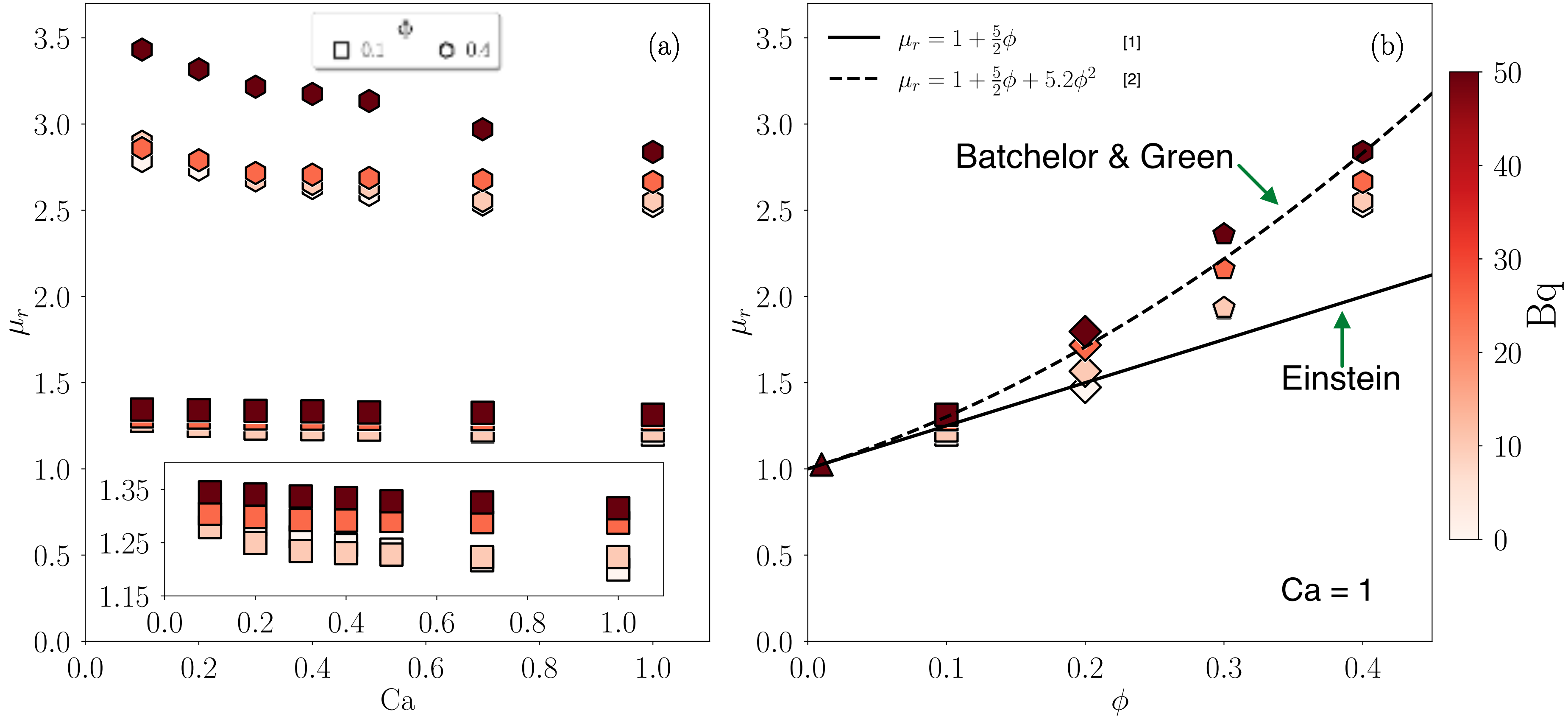
Rheology: **relative viscosity**

Suspension viscosity → Particle stress →

$$\mu_r = \frac{\mu_s}{\mu} = 1 + \frac{\Sigma^p_{xz}}{\dot{\gamma}\mu}$$

$$\text{Ca} = \frac{\mu\dot{\gamma}R}{k_s}$$

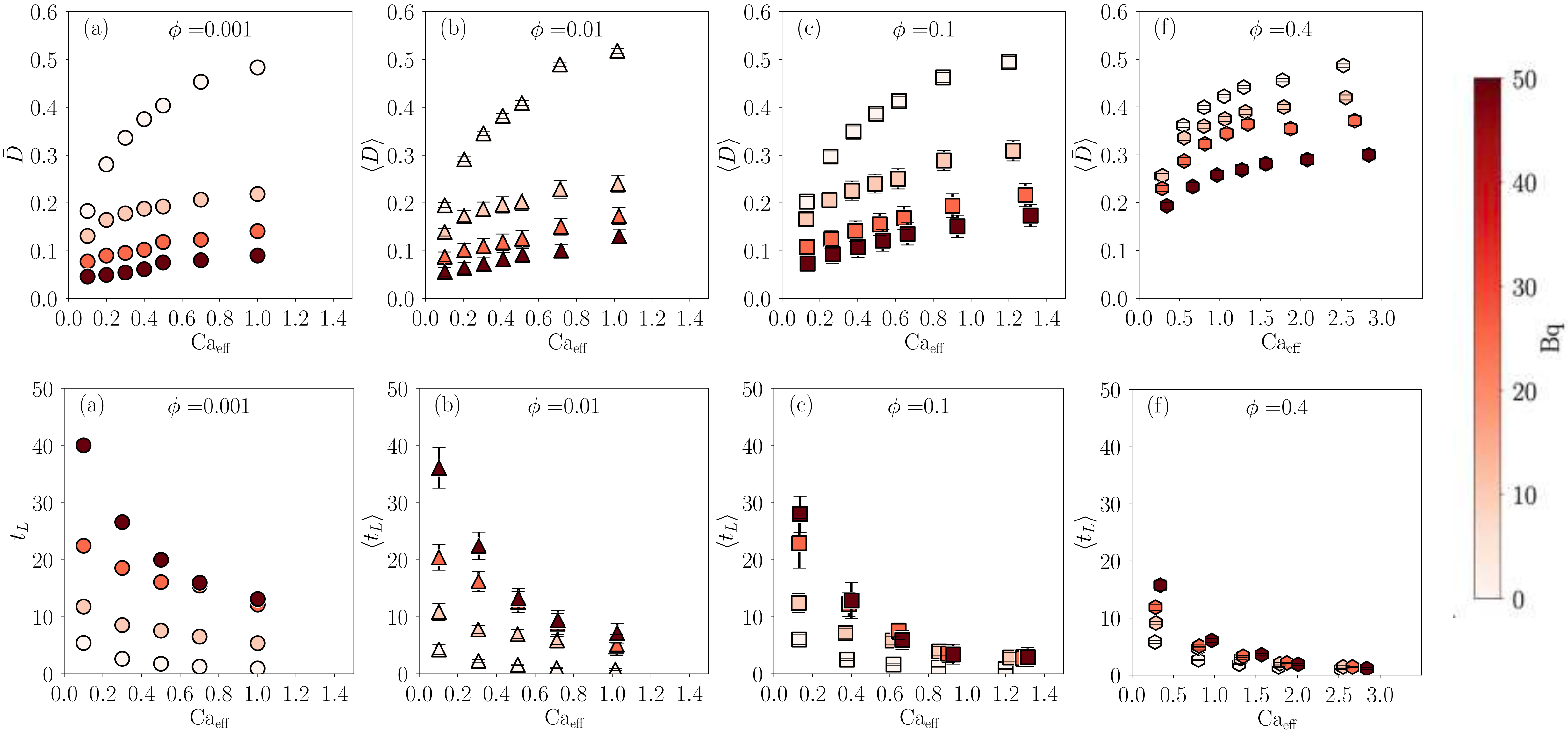
$$\text{Ca}_{\text{eff}} = \frac{\mu_s\dot{\gamma}R}{k_s} = \text{Ca}\mu_r$$



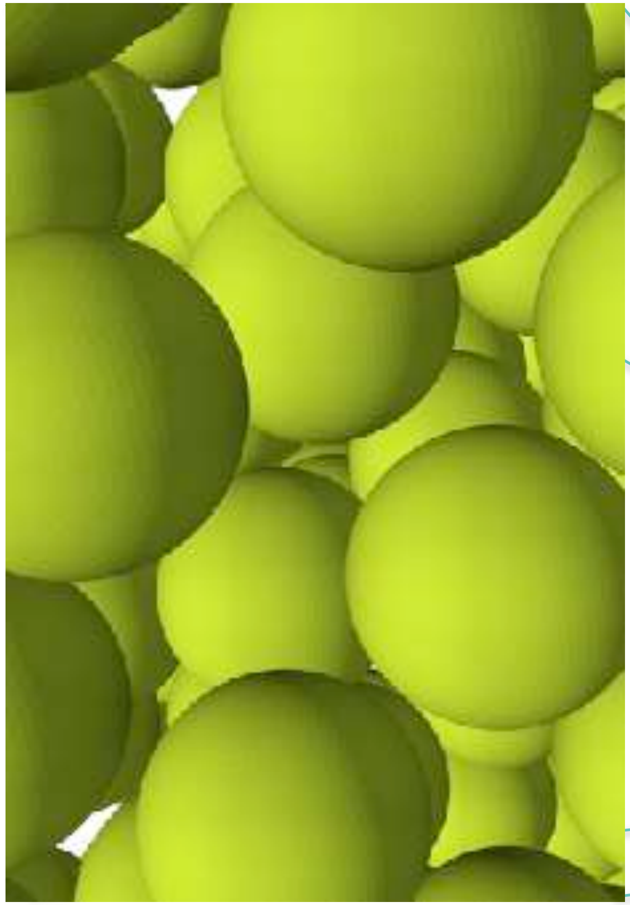
[1] Einstein, A. (1905). *Eine neue bestimmung der moleküldimensionen* (Doctoral dissertation).

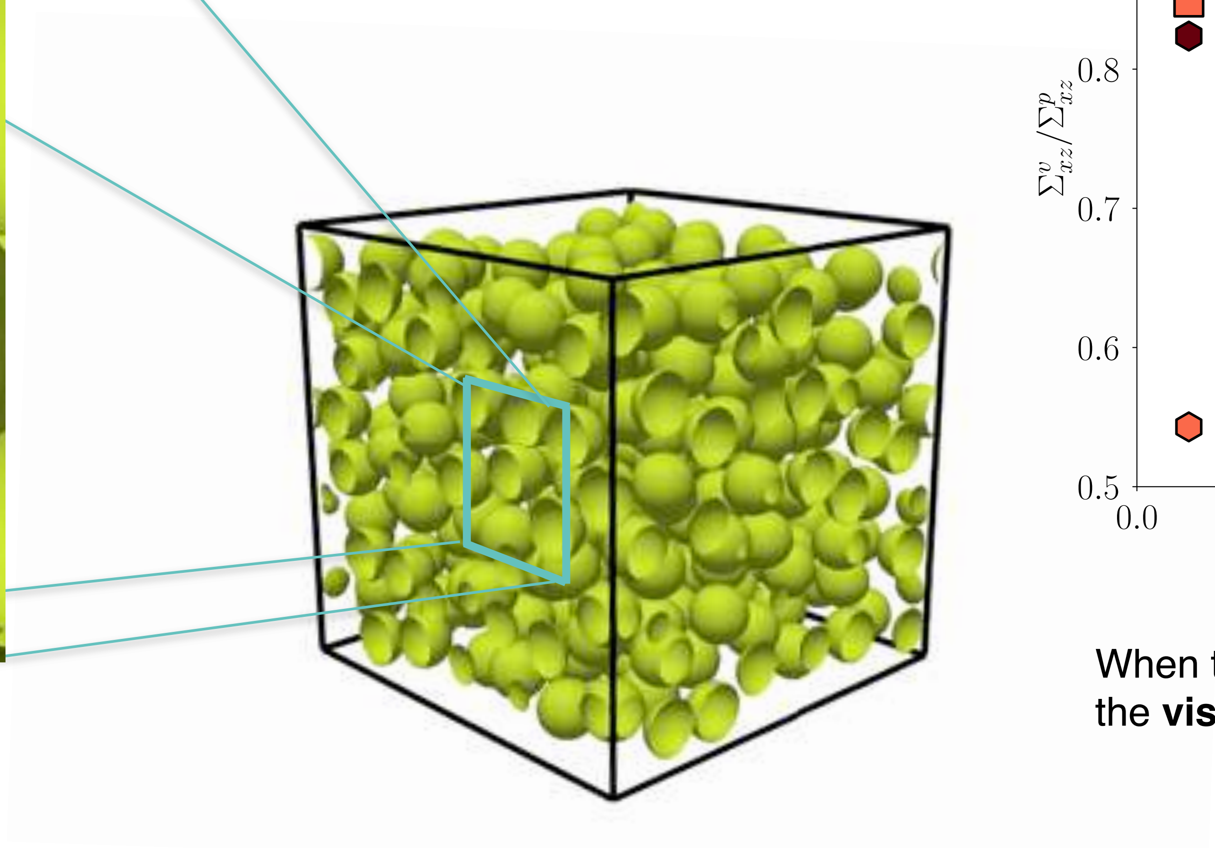
[2] Batchelor, G. K., & Green, J. (1972). The determination of the bulk stress in a suspension of spherical particles to order c2. *Journal of Fluid Mechanics*, *56*(3), 401-427.

Deformation and inclination angle



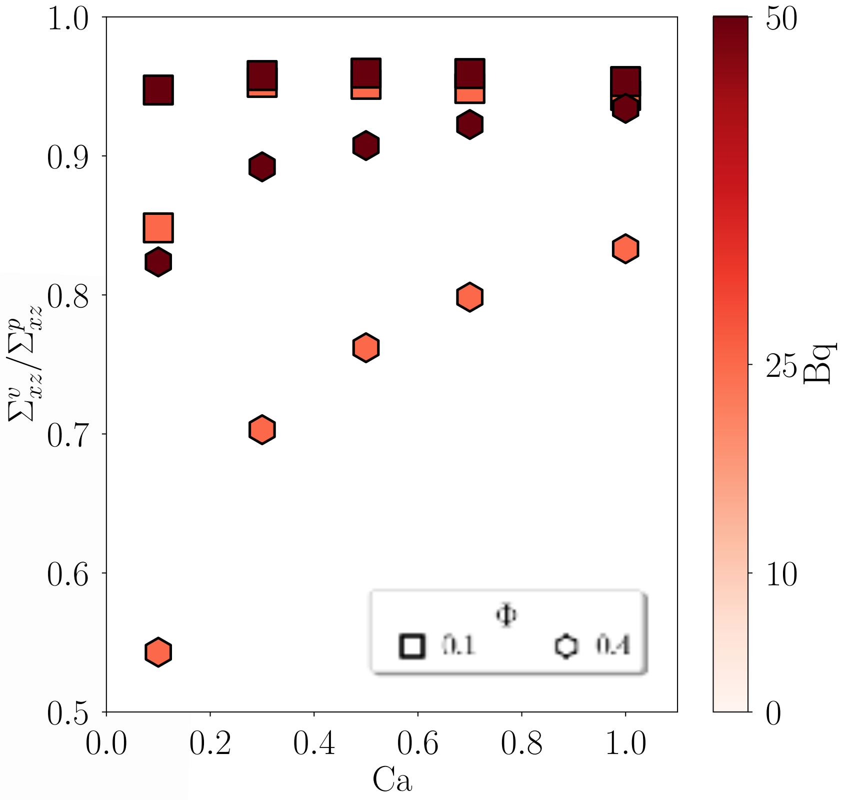
Membrane viscosity and volume fraction





Boussinesq-Scriven law

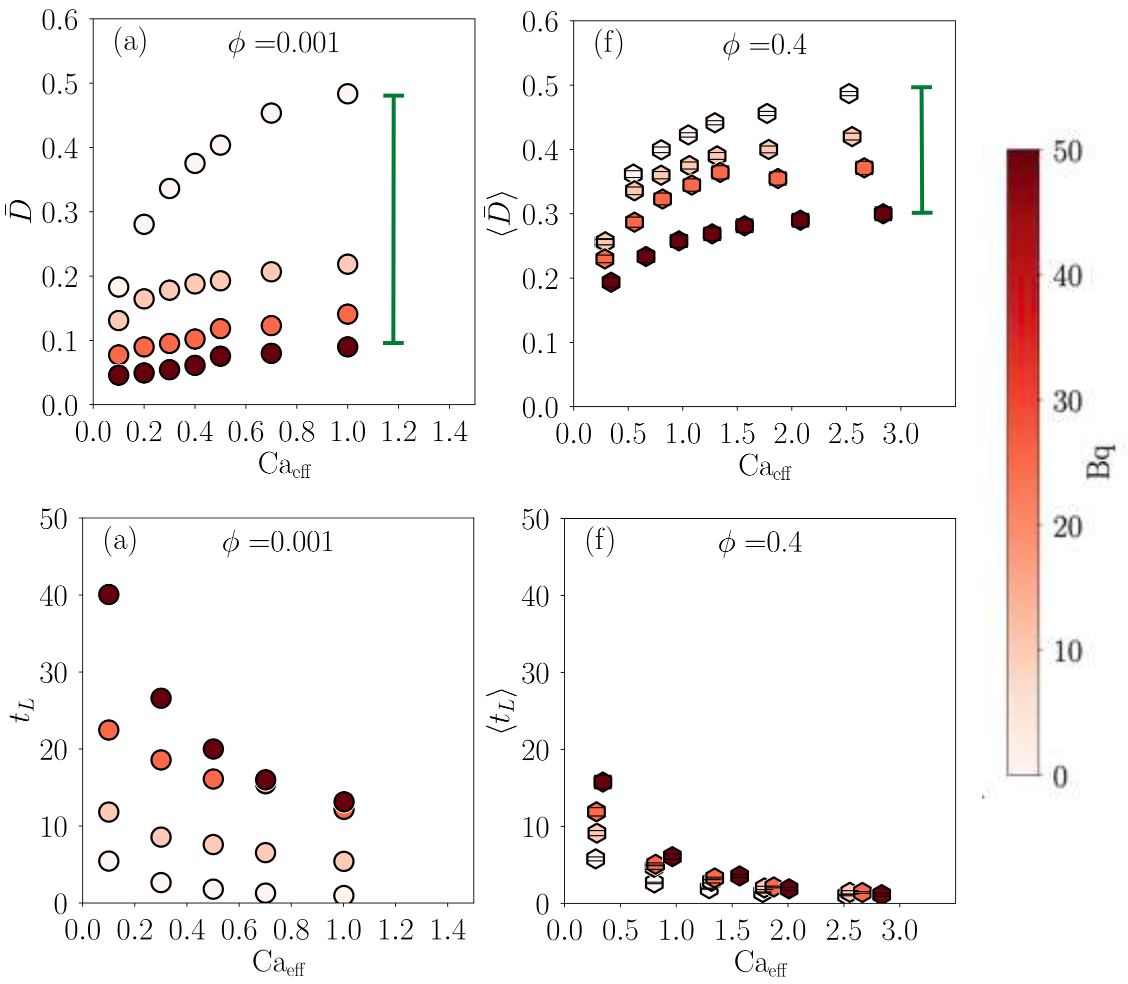
$$\tau^{v} = 2\mu_m \mathbf{E} = \mu_m[\nabla \mathbf{u} + \nabla \mathbf{u}^T]$$



When the **volume fraction increases**, the **viscous dissipation reduces**

What's the effect of membrane viscosity?

- For a **fixed value of ϕ,** the effect of Bq is to:
  - reduce the deformation
  - increase the loading time
- When ϕ **increases:**
  - the effect of Bq on D reduces
  - the effect of Bq on t_L almost vanishes

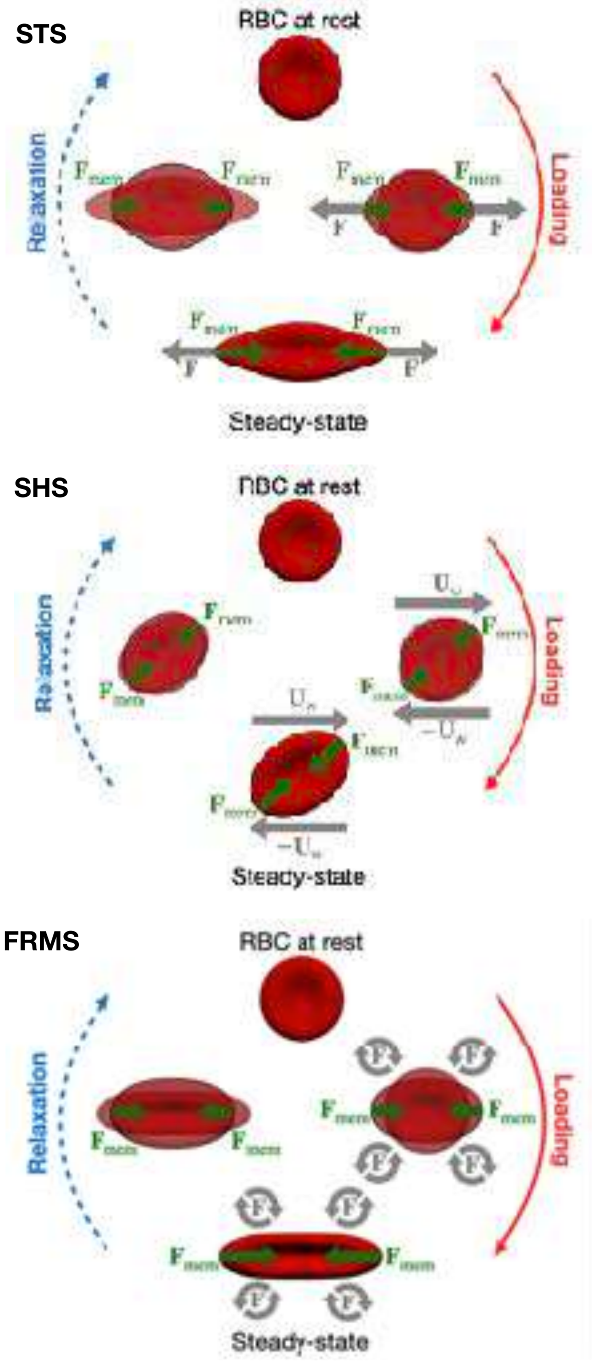


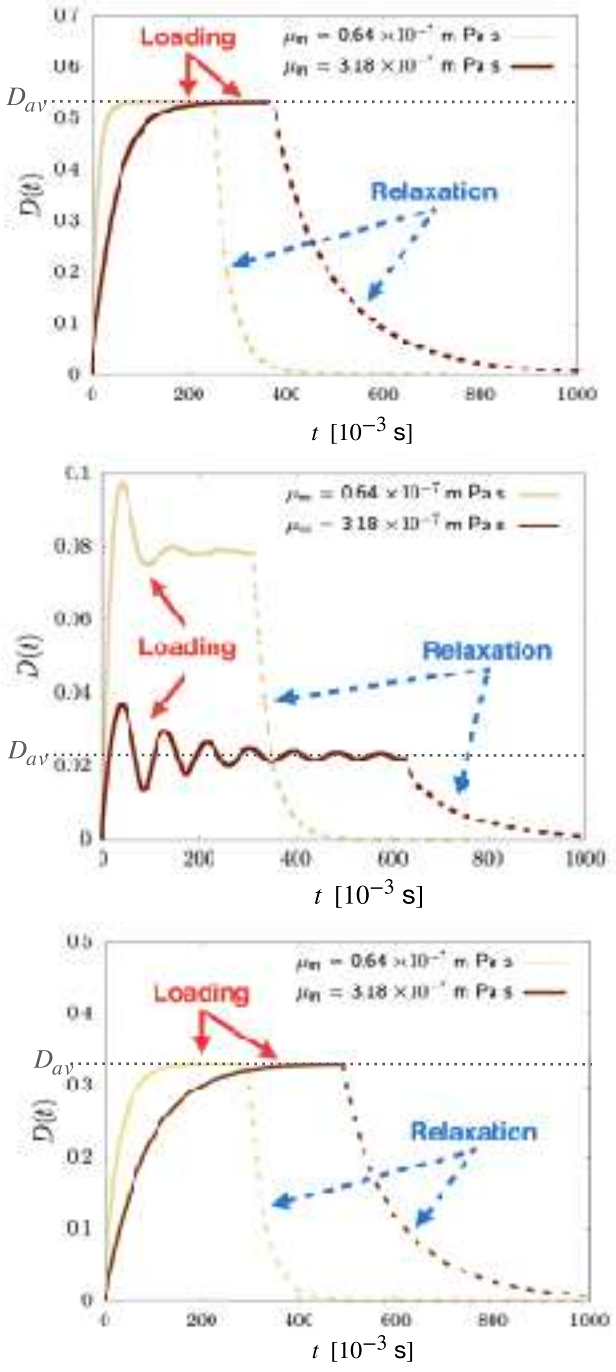
This work was supported by the Italian Ministry of University and Research (MUR) under the FARE programme, project “Smart-HEART”

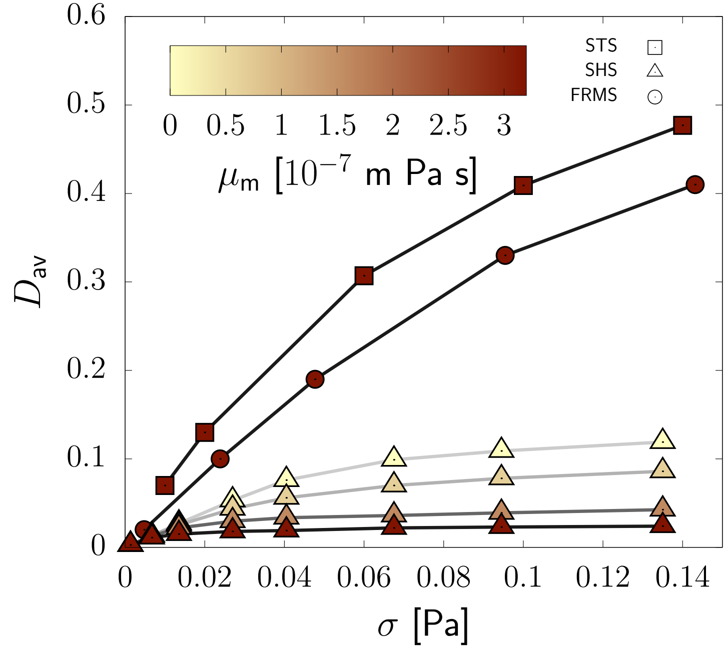
Thank you!

Guglietta, F., Pelusi, F., Sega, M., Aouane, O., & Harting, J. (2023).
Suspensions of viscoelastic capsules: Effect of membrane viscosity on transient dynamics.
Journal of Fluid Mechanics, **971**, A13. doi:10.1017/jfm.2023.694

BACKUP SLIDES



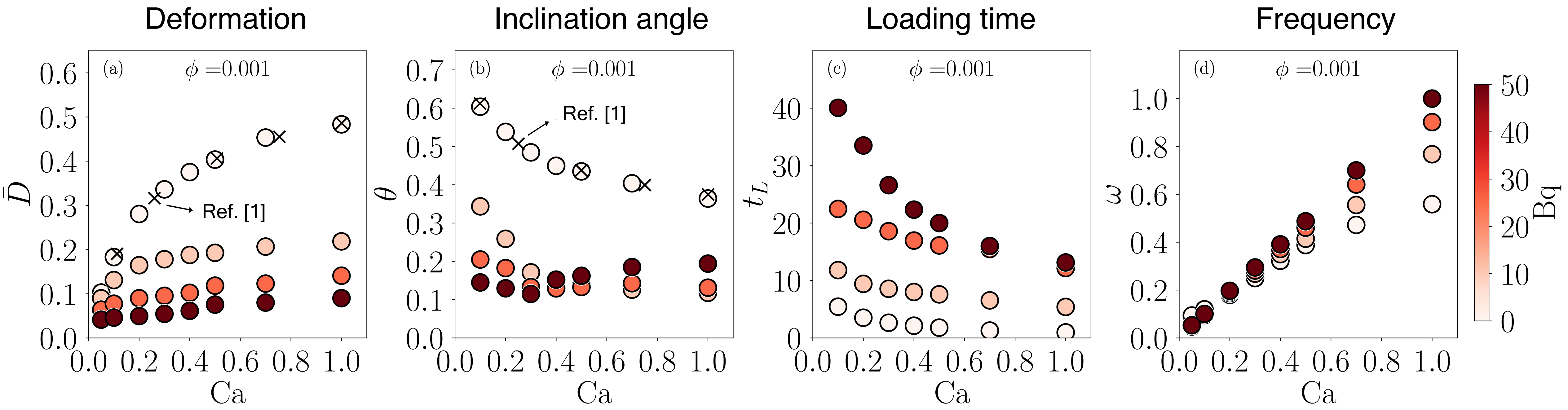




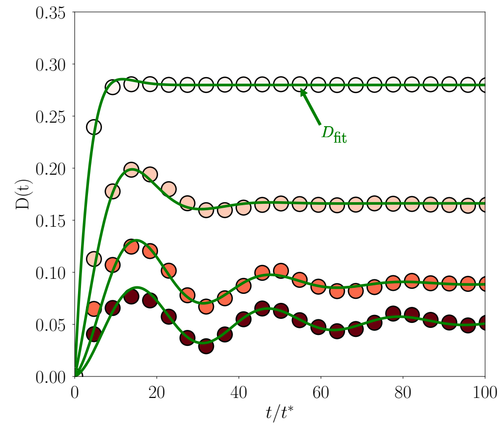
- **The steady value of deformation** may depend on the value of **membrane viscosity**
- **The rotation** of the membrane **dissipates** part of the energy otherwise used to **deform** the membrane.

F. Guglietta et al., "Loading and relaxation dynamics for a red blood cell", *Soft matter*, 17, 5978-5990, 2021.

Single capsule in simple shear flow

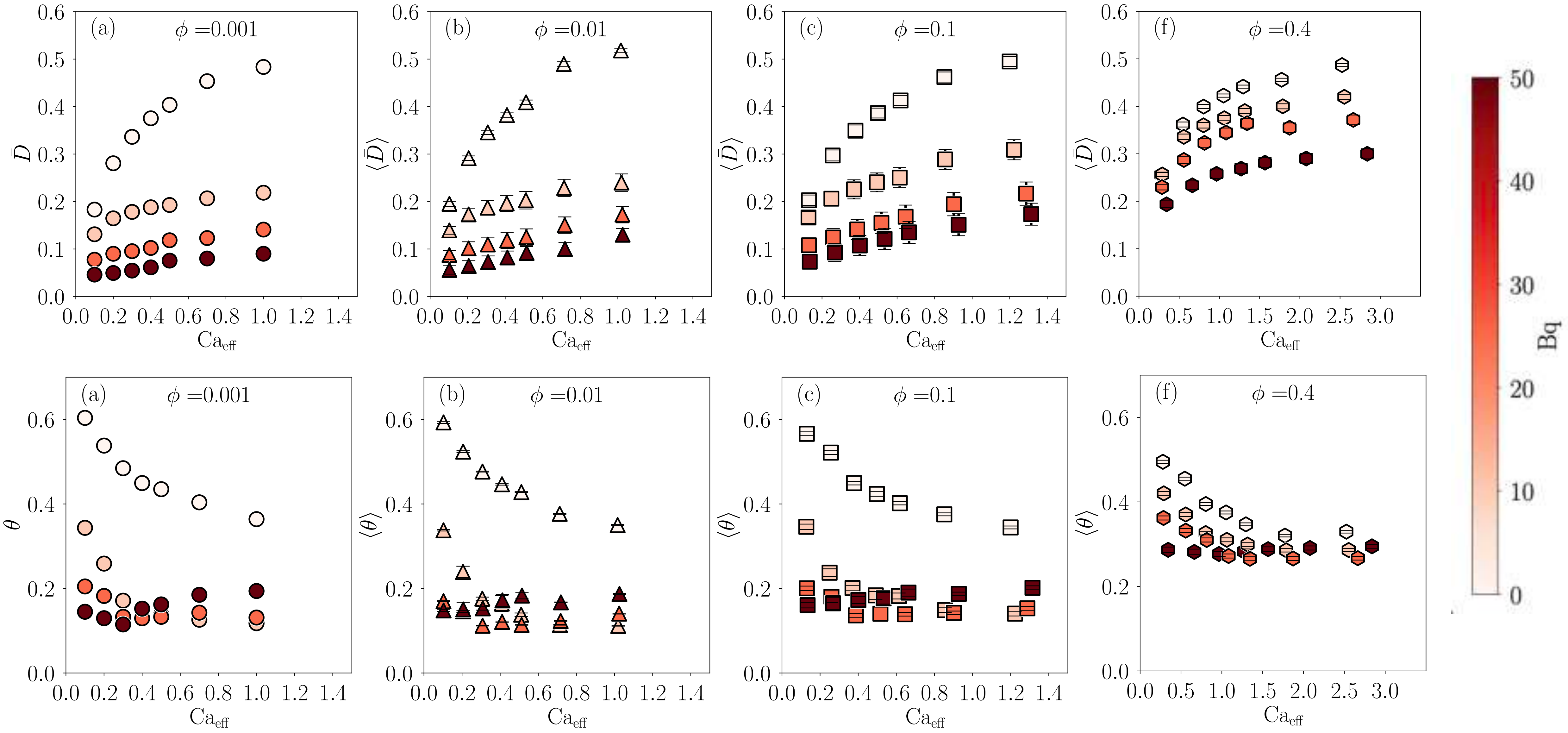


$$D_{\text{fit}}\left(\frac{t}{t^*}\right) = \bar{D}\left[1 - \exp\left(-\frac{t}{t^* t_L}\right)\cos\left(\omega\frac{t}{t^*}\right)\right]$$

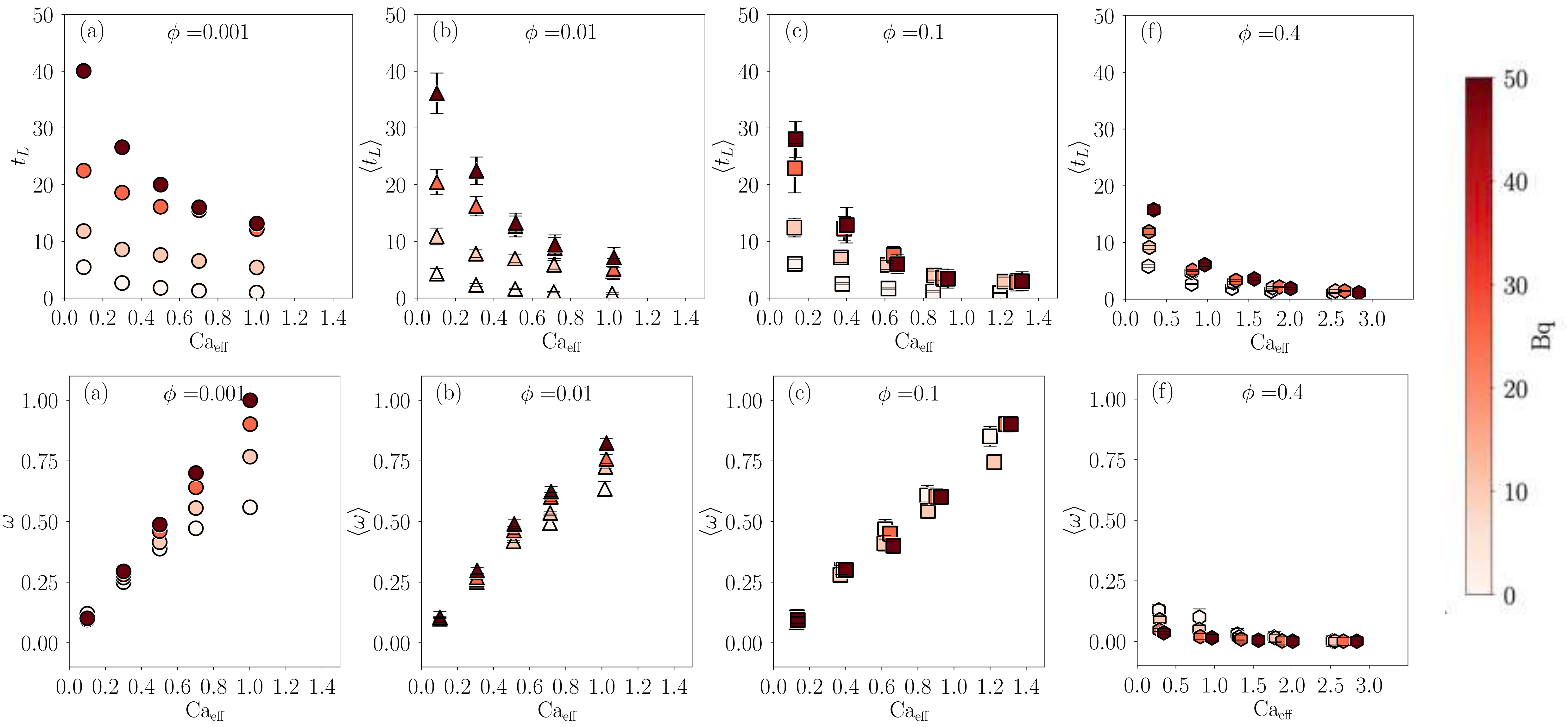


(1) Aouane, O., Scagliarini, A., & Harting, J. (2021). Structure and rheology of suspensions of spherical strain-hardening capsules. Journal of Fluid Mechanics, 911.

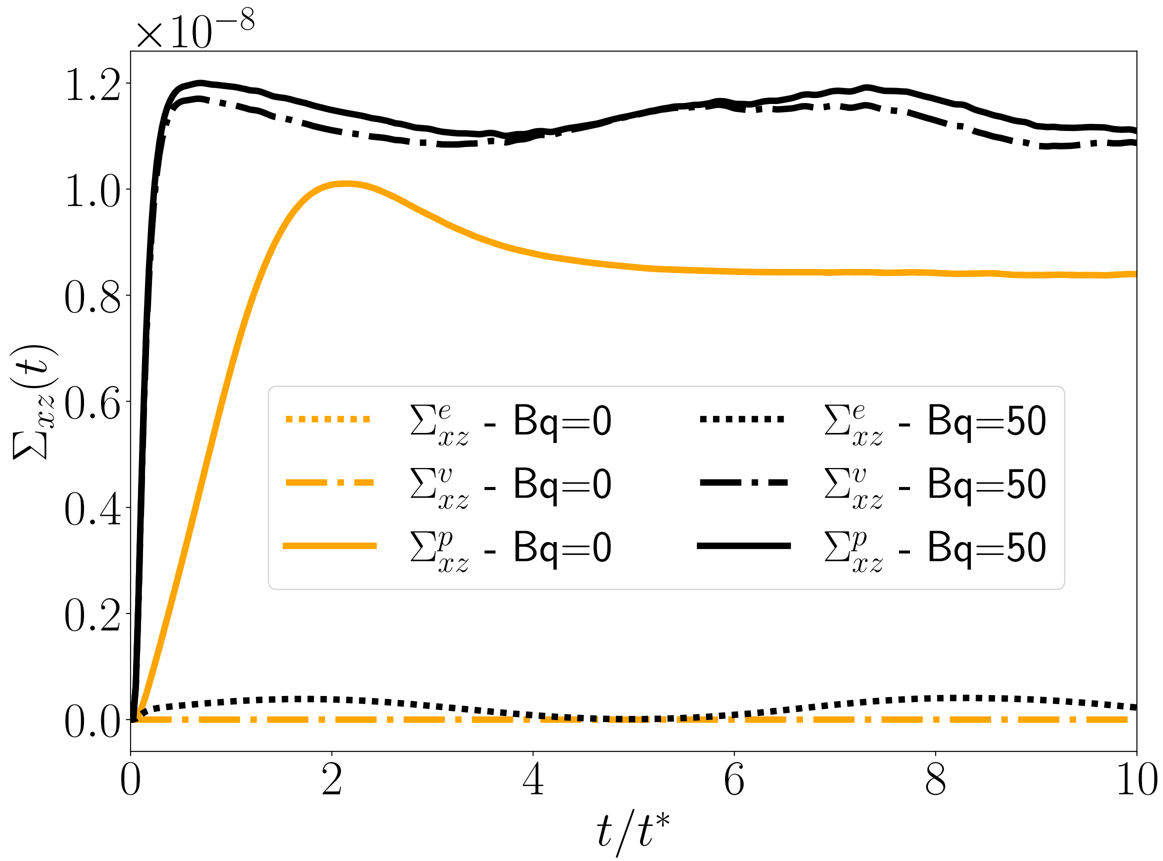
Deformation and inclination angle



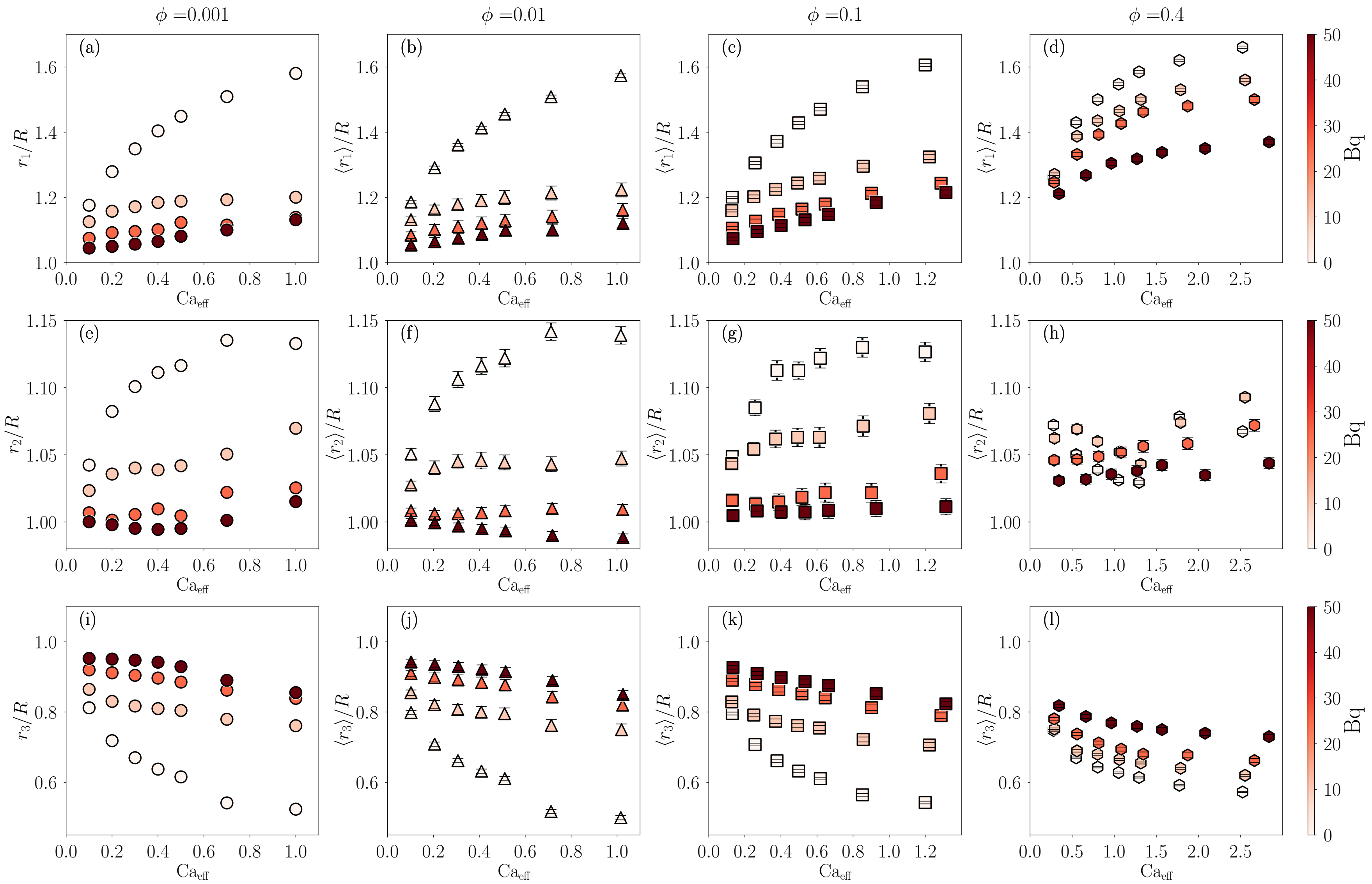
Loading time and frequency



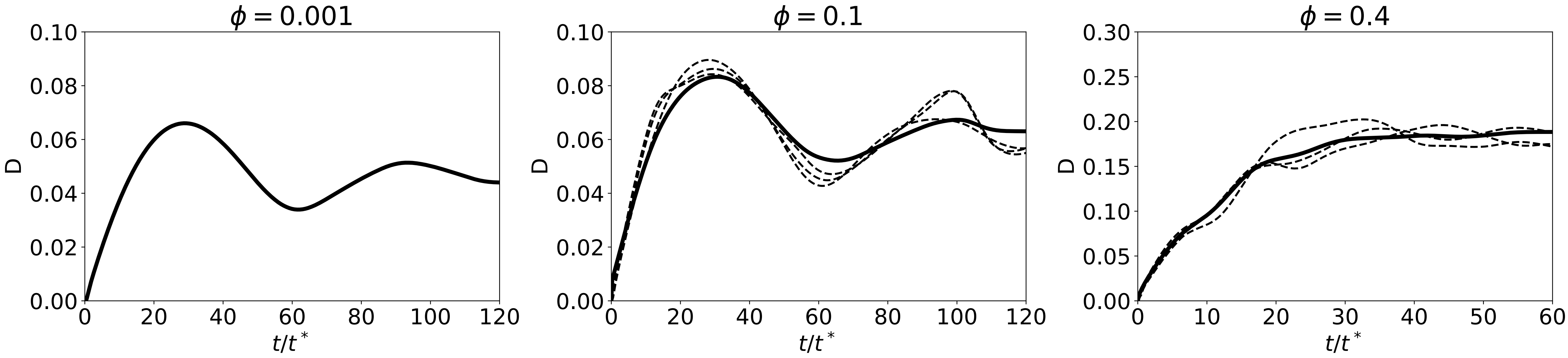
Single capsule: particle stress

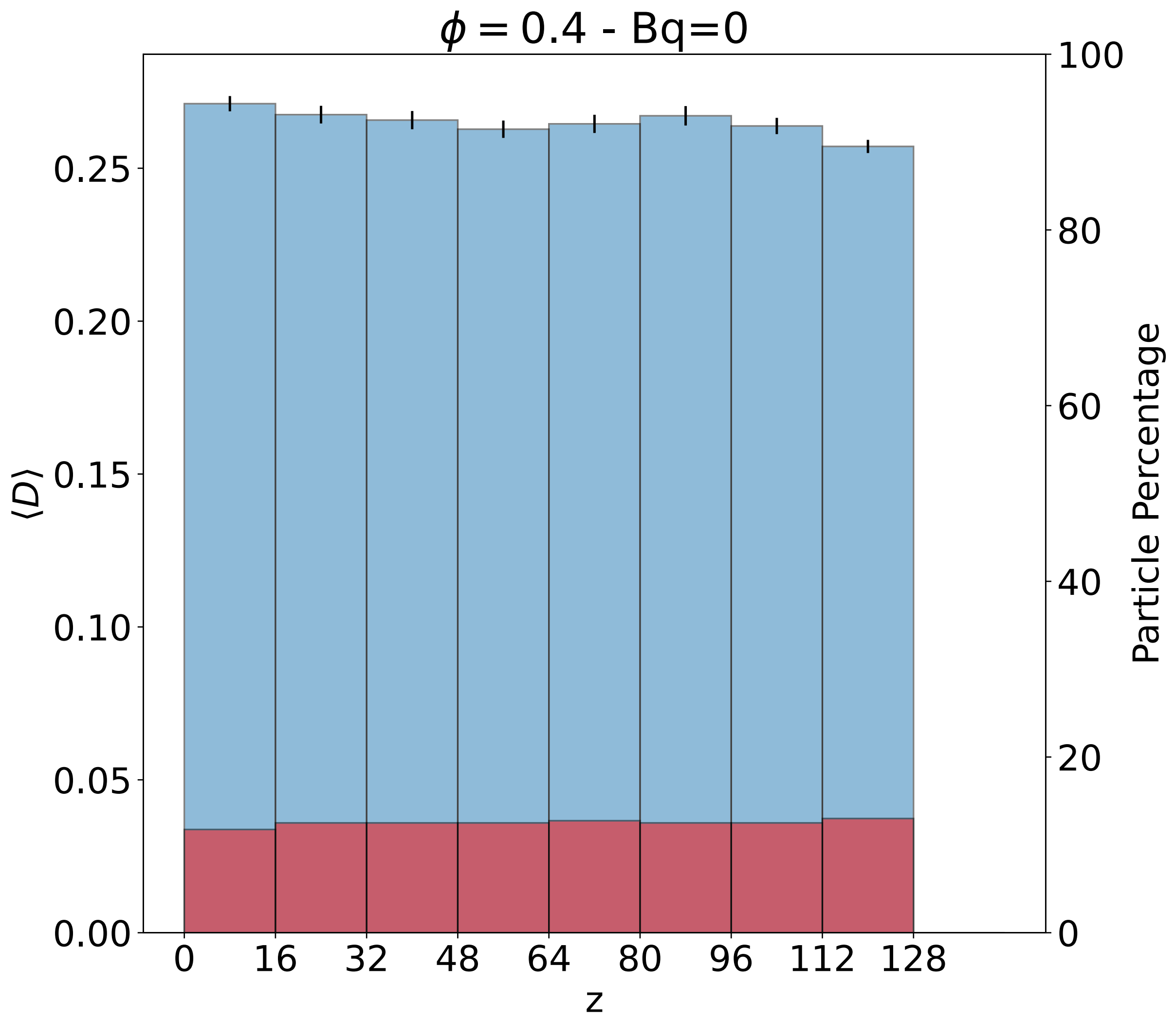


Radii

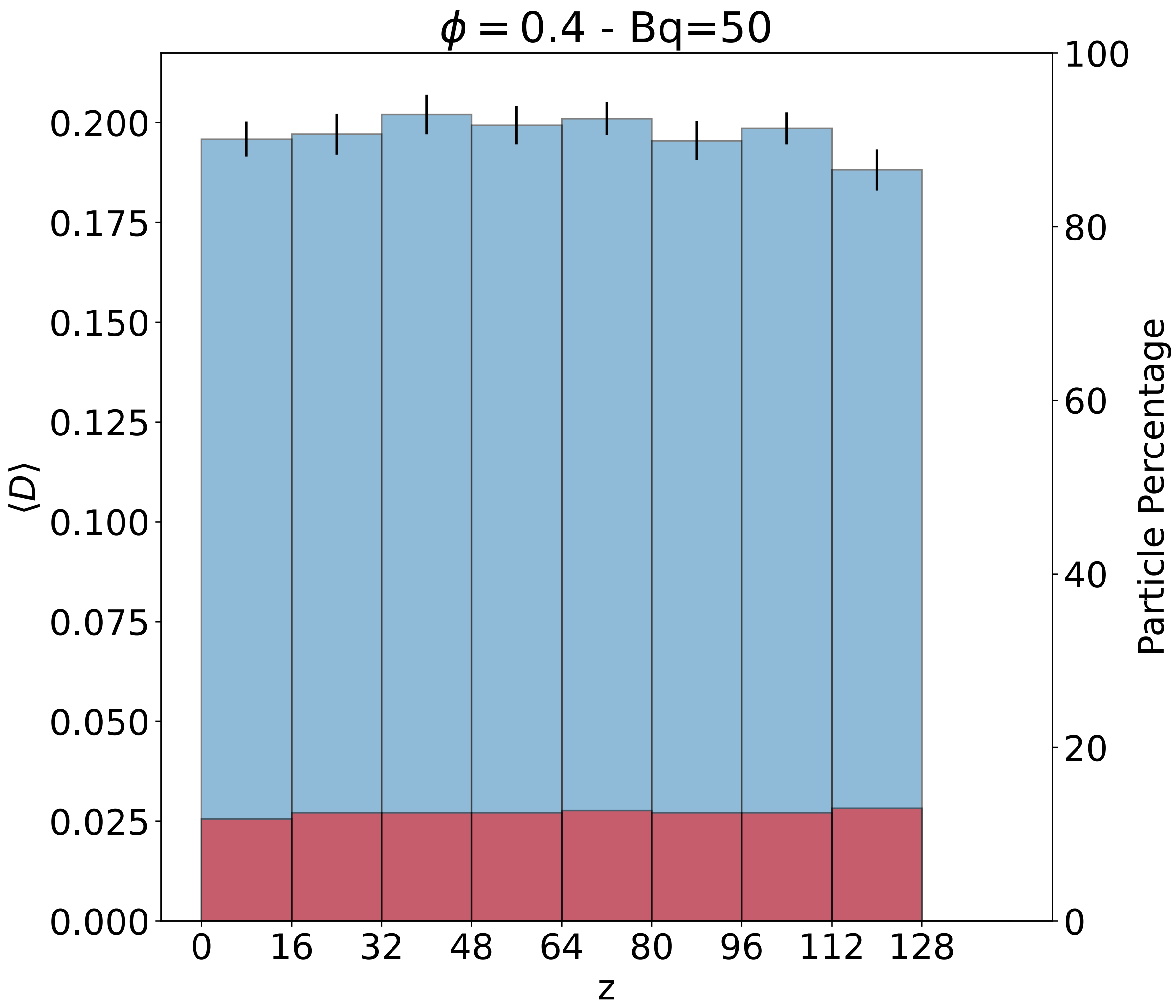


ϕ =0.001
ϕ =0.01
ϕ =0.1
ϕ =0.4
(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k) (l)
r_1/R
$\langle r_1\rangle/R$
r_2/R
$\langle r_2\rangle/R$
r_3/R
$\langle r_3\rangle/R$
$\mathrm{Ca_{eff}}$
Bq


$\phi = 0.001$
$\phi = 0.1$
$\phi = 0.4$
D
t/t^*
0.00
0.02
0.04
0.06
0.08
0.10
0.05
0.15
0.20
0.25
0.30
0
10
20
30
40
50
60
80
100
120



$\phi = 0.4$ - Bq=0
$\langle D \rangle$
0.00
0.05
0.10
0.15
0.20
0.25
Particle Percentage
0
20
40
60
80
100
0 16 32 48 64 80 96 112 128
z



$\phi = 0.4$ - Bq=50
$\langle D \rangle$
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
Particle Percentage
0
20
40
60
80
100
0 16 32 48 64 80 96 112 128
z

Initialisation